8-3: Growing, Growing, Growing

Unit Goals, Focus Questions, and Mathematical Reflections

Unit Goals

Exponential Functions Explore problem situations in which two or more variables have an exponential relationship to each other

- Identify situations that can be modeled with an exponential function
- Identify the pattern of change (growth/decay factor) between two variables that represent an exponential function in a situation, table, graph, or equation
- Represent an exponential function with a table, graph, or equation
- Make connections among the patterns of change in a table, graph, and equation of an exponential function
- Compare the growth/decay rate and growth/decay factor for an exponential function and recognize the role each plays in an exponential situation
- Identify the growth/decay factor and initial value in problem situations, tables, graphs, and equations that represent exponential functions
- Determine whether an exponential function represents a growth (increasing) or decay (decreasing) pattern, from an equation, table, or graph that represents an exponential function
- Determine the values of the independent and dependent variables from a table, graph, or equation of an exponential function
- Use an exponential equation to describe the graph and table of an exponential function
- Predict the *y*-intercept from an equation, graph, or table that represents an exponential function
- Interpret the information that the *y*-intercept of an exponential function represents
- Determine the effects of the growth (decay) factor and initial value for an exponential function on a graph of the function
- Solve problems about exponential growth and decay from a variety of different subject areas, including science and business, using an equation, table, or graph
- Observe that one exponential equation can model different contexts
- Compare exponential and linear functions

Equivalence Develop understanding of equivalent exponential expressions

- Write and interpret exponential expressions that represent the dependent variable in an exponential function
- Develop the rules for operating with rational exponents and explain why they work
- Write, interpret, and operate with numerical expressions in scientific notation
- Write and interpret equivalent expressions using the rules for exponents and operations
- Solve problems that involve exponents, including scientific notation

8-3 Growing, Growing, Growing: Focus Questions (FQ) and Mathematical Reflections

Investigation 1 Exponential Growth	**Investigation 2** Examining Growth Patterns	**Investigation 3** Growth Factors and Growth Rates	**Investigation 4** Exponential Decay	**Investigation 5** Patterns with Exponents
Problem 1.1 **Making Ballots: Introducing Exponential Functions** FQ: What are the variables in this situation and how are they related? **Problem 1.2** **Requesting a Reward: Representing Exponential Functions** FQ: In what ways are the relationships represented in a chessboard and ballot-cutting situations similar? Different? **Problem 1.3** **Making a New Offer: Growth Factors** FQ: How does the growth pattern for an exponential function show up in a table, graph, or equation that represents the function and how does it compare to the growth pattern in a linear function?	**Problem 2.1** **Killer Plant Strikes Lake Victoria: y-intercepts Other Than 1** FQ: What information do you need to write an equation that represents an exponential function? **Problem 2.2** **Growing Mold: Interpreting Equations for Exponential Functions** FQ: How is the growth factor and initial population for an exponential function represented in an equation that represents the function? **Problem 2.3** **Studying Snake Populations: Interpreting Graphs of Exponential Functions** FQ: How is the growth factor and initial population for an exponential function represented in a graph that represents the function?	**Problem 3.1** **Reproducing Rabbits: Fractional Growth Patterns** FQ: How is the growth factor in this Problem similar to that in the previous Problems? How is it different? **Problem 3.2** **Investing for the Future: Growth Rates** FQ: How are the growth factor and growth rate for an exponential function related? When might you use each in an exponential growth pattern? **Problem 3.3** **Making a Difference: Connecting Growth Rate and Growth Factor** FQ: How does the initial population affect the growth patterns in an exponential function?	**Problem 4.1** **Making Smaller Ballots: Introducing Exponential Decay** FQ: How does the pattern of change in this situation compare to growth patterns you have studied in previous Problems? How does the difference show up in a table, graph, and equation? **Problem 4.2** **Fighting Fleas: Representing Exponential Decay** FQ: How can you recognize an exponential decay function from a contextual setting, table, graph, and equation that represents the function? **Problem 4.3** **Cooling Water: Modeling Exponential Decay** FQ: How can you find the initial population and decay factor for an exponential decay relationship?	**Problem 5.1** **Looking for Patterns Among Exponents** FQ: What patterns did you observe in the table of powers? **Problem 5.2** **Rules of Exponents** FQ: What are several rules for working with exponents and why do they work? **Problem 5.3** **Extending the Rules of Exponents** FQ: How are the rules for integral exponents related to rational exponents? How are the rules for exponents useful in writing equivalent expressions with exponents? **Problem 5.4** **Operations with Scientific Notation** FQ: How does scientific notation help to solve problems? **Problem 5.5** **Revisiting Exponential Functions** FQ: What are the effects of a and b on the graph of $y = a(b^x)$, $b \neq 0$.
Mathematical Reflections 1. Describe an exponential growth pattern. Include key properties such as growth factors. 2. How are exponential functions similar to and different from the linear functions you worked with in earlier Units?	**Mathematical Reflections** 1. How can you use a table, a graph, and an equation that represent an exponential function to find the y-intercept and growth factor for the function? Explain. 2. How can you use the y-intercept and growth factor to write an equation that represents an exponential function? Explain. 3. How would you change your answers to Questions 1 and 2 for a linear function?	**Mathematical Reflections** 1. Suppose you know the initial value for a population and the yearly growth rate. 1a. How can you determine the population several years from now? 1b. How is a growth rate related to the growth factor for the population? 1c. How can you use this information to write an equation that models the situation? 2. Suppose you know the initial value for a population and the yearly growth factor. 2a. How can you determine the population several years from now? 2b. How can you determine the yearly growth rate? 3. Suppose you know the equation that represents the exponential function relating the population p and the number of years n. How can you determine the doubling time for the population?	**Mathematical Reflections** 1. How can you recognize an exponential decay pattern from the following? 1a. a table of data 1b. a graph 1c. an equation 2. How are exponential growth functions and exponential decay functions similar? How are they different? 3. How are exponential decay functions and decreasing linear functions similar? How are they different?	**Mathematical Reflections** 1a. Describe some of the rules for operating with exponents. 1b. What is scientific notation? What are its practical applications? 2. Describe the effects of a and b on the graph of $y = a(b^x)$. 3. Compare exponential and linear functions. Include in your comparison information about their patterns of change, y-intercepts, whether the function is decreasing or increasing, and any other information you think is important. Include examples of how they are useful.